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# Derivation of linewidths for optical transitions in quantum wells due to longitudinal optical phonon scattering

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## Abstract

Utilizing the quantum statistical operator algebra technique, we derive a new absorption coefficient formula for the optical transition in a system of electrons interacting with longitudinal optical phonons in a quantum well. The electron and phonon distribution functions are included in the lineshape function. So we can explain the phonon absorptions and emissions in an organized way for all the electron transition processes. The numerical results show that the width decreases with the well width and increases with the temperature.

## 1. Introduction

With the recent progress in modern crystal growth techniques [1], such as molecular beam epitaxy (MBE) and metal–organic chemical vapour deposition (MOCVD), it has become possible to grow semiconductors of different atomic compositions on top of a semiconductor substrate with monolayer precision. Among those low-dimensional systems, semiconductor optoelectronic devices, such as laser diodes, optical waveguides, and photodetectors, have important applications in optical communication systems. In order to understand the physics and the operational characteristics of these optoelectronic devices, we need to develop a quantum statistical method.

Optical spectral features such as gain and spontaneous emission are the most basic properties of semiconductor lasers [2–11]. Among the many factors determining an optical spectrum, e.g., the carrier distribution in the energy bands, the matrix elements for the transitions between electrons and holes, and the rate of relaxation of carriers due to various scattering processes, we are interested in the last one.

There are many quantum statistical methods applied in these areas [2–5]. The present group has introduced some many-body theories for transport phenomena [12–14]. These

theories were obtained by applying several projection techniques [15–17] and were applied in the calculation of the linewidth in magneto-optical transitions in semiconductors. In this paper, by utilizing one such theory [14], we will introduce a new theoretical method for obtaining the relaxation rate for a system of electrons interacting with phonons.

## 2. Formal solution of the Liouville equation

When a time-dependent external electric field

$$\mathbf{E}(t) = E_0 \exp[-i(\omega t - kx)]\hat{\mathbf{z}} + \text{complex conjugate} \quad (2.1)$$

is applied in the  $z$ -direction, the total Hamiltonian  $H(t)$  is given by

$$H(t) = H_{eq} + H_{int}(t). \quad (2.2)$$

Here  $H_{eq}$  is the Hamiltonian of a system of many electrons interacting with phonons (or electrons) in thermodynamic equilibrium and  $H_{int}(t)$  is the time-dependent part given by

$$H_{int}(t) = -\mathbf{P} \cdot \mathbf{E}(t) = -\sum_{\alpha\beta} (\mathbf{R})_{\alpha\beta} a_{\alpha}^{\dagger} a_{\beta} E_0 \exp(-i\omega t) + \text{complex conjugate} \quad (2.3)$$

where  $\mathbf{P} = \sum_{\alpha\beta} (\mathbf{R})_{\alpha\beta} a_{\alpha}^{\dagger} a_{\beta}$  is the polarization vector,  $\mathbf{R}$  is the dipole moment operator,  $a_{\alpha}^{\dagger}$  ( $a_{\alpha}$ ) denotes the creation (annihilation) operator for an electron in the state  $|\alpha\rangle$  with energy  $E_{\alpha}$  and  $(X)_{\alpha\beta} \equiv \langle\alpha|X|\beta\rangle$ .

We assume that when the time-dependent electric field is applied to the system, the density operator changes as

$$\rho(t) = \rho_{eq} + \rho_{int}(t) \quad (2.4)$$

where  $\rho_{eq}$  is the equilibrium density operator and  $\rho_{int}(t)$  is the term for the perturbation by the time-dependent electric field. Using the Liouville equation

$$i\hbar \frac{\partial \rho(t)}{\partial t} = [H(t), \rho(t)] = L(t)\rho(t) \quad (2.5)$$

we can obtain

$$i\hbar \frac{\partial \rho_{int}(t)}{\partial t} = [H_{eq}, \rho_{int}(t)] + [H_{int}(t), \rho_{eq}] + [H_{int}(t), \rho_{int}(t)] \quad (2.6)$$

where we have used the facts that  $\partial \rho_{eq}/\partial t = 0$  and  $[H_{eq}, \rho_{eq}] = 0$ . In equation (2.5),  $L(t)$  is the Liouville operator defined as  $L(t)X \equiv H(t)X - XH(t)$  for a arbitrary operator  $X$ .  $L(t)$  can be split into two parts, for the sake of calculational convenience:  $L(t) = L_{eq} + L_{int}(t)$ , where  $L_{eq}$  and  $L_{int}$ , respectively, correspond to  $H_{eq}$  and  $H_{int}$ .

In order to obtain  $\rho_{int}(t)$ , we define the density operator in the Dirac picture as

$$\rho_{int}^D(t) \equiv \exp(iH_{eq}t/\hbar)\rho_{int}(t)\exp(-iH_{eq}t/\hbar). \quad (2.7)$$

Differentiating equation (2.7) and considering equation (2.6), we obtain

$$i\hbar \frac{\partial \rho_{int}^D(t)}{\partial t} = \exp(iL_{eq}t/\hbar)L_{int}(t)\rho_{eq} + \exp(iL_{eq}t/\hbar)L_{int}(t)\rho_{int}(t). \quad (2.8)$$

Integrating this differential equation from  $t = -\infty$  to  $t$  subject to the initial condition  $\rho_{int}^D(-\infty) = 0$ , we can obtain a solution to this equation:

$$i\hbar \rho_{int}^D(t) = \int_{-\infty}^t du \exp(-iL_{eq}u/\hbar)[L_{int}(u)\rho_{eq}] + \int_{-\infty}^t du \exp(iL_{eq}u/\hbar)[L_{int}(u)\rho_{int}(u)]. \quad (2.9)$$

Now, on inserting equation (2.7) into (2.9) and replacing  $t - u$  by  $t_1$ , we get

$$\begin{aligned} \rho_{int}(t) &= \frac{1}{i\hbar} \int_0^\infty dt_1 \exp(-iL_{eq}t_1/\hbar)[L_{int}(t - t_1)\rho_{eq}] \\ &\quad + \frac{1}{i\hbar} \int_0^\infty dt_1 \exp(-iL_{eq}t_1/\hbar)[L_{int}(t - t_1)\rho_{int}(t - t_1)]. \end{aligned} \quad (2.10)$$

Finally we obtain by iteration

$$\begin{aligned} \rho_{int}(t) &= \sum_{n=1}^{\infty} \frac{1}{(i\hbar)^n} \int_0^\infty dt_1 \int_0^\infty dt_2 \cdots \int_0^\infty dt_n \exp(-iL_{eq}t_1/\hbar)L_{int}(t - t_1) \\ &\quad \times \exp(-iL_{eq}t_2/\hbar)L_{int}(t - t_1 - t_2) \\ &\quad \times \cdots \times \exp(-iL_{eq}t_n/\hbar)L_{int}(t - t_1 - \cdots - t_n)\rho_{eq} \\ &\equiv \rho^{(1)}(t) + \rho^{(2)}(t) \cdots \rho^{(n)}(t) \end{aligned} \quad (2.11)$$

where  $L_{int}(t)$  is involved  $n$  times in  $\rho^{(n)}(t)$ .

Using equation (2.11), we obtain the ensemble average of the polarization operator  $P$  as follows:

$$P(t) = \sum_{n=1}^{\infty} P^{(n)}(t) = \sum_{n=1}^{\infty} T_R\{\rho^{(n)}(t)P\} \quad (2.12)$$

where  $T_R$  means the many-body trace.

Here we are interested in the linear absorption spectra. Thus we only have to take the first order of  $P(t)$ , which is given by

$$\begin{aligned} P^{(1)}(t) &= T_R\{P\rho^{(1)}(t)\} \\ &= \frac{-1}{i\hbar} \sum_{\alpha\beta} (R)_{\alpha\beta} \int_0^\infty ds T_R\{\exp[is(L_{eq} + \hbar\bar{\omega})/\hbar]P[a_\alpha^+a_\beta, \rho_{eq}]\} E_0 e^{-i\omega t} \end{aligned}$$

or

$$P^{(1)}(t) \equiv \chi(\omega) E_0 e^{i\omega t} \quad (2.13)$$

where  $\bar{\omega} \equiv \omega + ia$  ( $a \rightarrow 0^+$ ). The first-order susceptibility tensor is defined by

$$\chi(\omega) \equiv - \sum_{\alpha\beta} (R)_{\alpha\beta} \sum_{\gamma\delta} (R)_{\gamma\delta} A_{\alpha\beta}(\bar{\omega}). \quad (2.14)$$

Here

$$A_{\alpha\beta}(\bar{\omega}) \equiv T_R\{\rho_{eq}[(\hbar\bar{\omega} + L_{eq})^{-1}a_\gamma^+a_\delta, a_\alpha^+a_\beta]\} \quad (2.15)$$

where we used  $T_R\{A[B, C]\} = T_R\{C[A, B]\}$ . Equation (2.14) will be used in deriving the absorption coefficient later.

### 3. Review of absorption coefficient in linear scheme

The optical gain coefficient  $G$  is given by [3, 4]

$$G = \frac{\omega}{n_r} \sqrt{\frac{\mu_0}{\epsilon_0}} \frac{1}{E_0 T_0} \text{Im} \int_0^{T_0} P(t) e^{i\omega t} dt \quad (3.1)$$

where  $\omega$  is the angular frequency of the incident electric field,  $n_r$  is the refractive index,  $\mu_0$  and  $\epsilon_0$  are the permeability and permittivity of the vacuum, respectively,  $T_0$  is the period of the light wave and the notation ‘Im’ means the imaginary part.

Inserting equation (2.13) into (3.1), we obtain

$$G = -\frac{\omega}{n_r} \sqrt{\frac{\mu_0}{\epsilon_0}} \operatorname{Im} \sum_{\alpha\beta} \sum_{\gamma\delta} (R)_{\alpha\beta} (R)_{\gamma\delta} A_{\alpha\beta}(\bar{\omega}). \quad (3.2)$$

Now, in order to calculate  $A_{\alpha\beta}(\omega)$  further, we define the projectors  $P$  and  $Q$  as

$$PX \equiv \frac{\langle X \rangle}{\langle a_\gamma^+ a_\delta \rangle} a_\gamma^+ a_\delta \quad (3.3)$$

$$Q \equiv 1 - P \quad (3.4)$$

where

$$\langle X \rangle \equiv T_R \{ \rho_{eq} [X, a_\alpha^+ a_\beta] \}. \quad (3.5)$$

Applying the identity  $1 = P + Q$  in the right-hand side of the Liouville operator  $L_{eq}$  in equation (2.15) as  $L_{eq} = L_{eq}(P+Q)$  and using the identity  $(A+B)^{-1} = A^{-1} - A^{-1}B(A+B)^{-1}$ , we obtain

$$(\hbar\bar{\omega} + L_{eq})^{-1} a_\gamma^+ a_\delta = a_\gamma^+ a_\delta / \hbar\bar{\omega} - (\hbar\bar{\omega} + L_{eq}Q)^{-1} L_{eq} a_\gamma^+ a_\delta A_1(\omega) / \langle a_\gamma^+ a_\delta \rangle \quad (3.6)$$

where we used  $Pa_\gamma^+ a_\delta = a_\gamma^+ a_\delta$  and  $Qa_\gamma^+ a_\delta = 0$ . Considering  $L_{eq}$  as  $L_{eq} = L_d + L_v$  in the second term on the right-hand side of equation (3.6), where  $L_d$  and  $L_v$  are the Liouville operators corresponding to the Hamiltonians  $H_d$  (diagonal part) and  $V$  (nondiagonal part), respectively, we obtain

$$\begin{aligned} (\hbar\bar{\omega} + L_{eq})^{-1} a_\gamma^+ a_\delta &= a_\gamma^+ a_\delta / \hbar\bar{\omega} - (E_\gamma - E_\delta) a_\gamma^+ a_\delta A_1(\omega) / \hbar\bar{\omega} \langle a_\gamma^+ a_\delta \rangle \\ &\quad - L_v a_\gamma^+ a_\delta A_1(\omega) / \hbar\bar{\omega} \langle a_\gamma^+ a_\delta \rangle + L_{eq} Q (\hbar\bar{\omega} + L_{eq} Q)^{-1} L_v a_\gamma^+ a_\delta A_1(\omega) / \hbar\bar{\omega} \langle a_\gamma^+ a_\delta \rangle. \end{aligned} \quad (3.7)$$

Here we used  $L_d a_\gamma^+ a_\delta = (E_\gamma - E_\delta) a_\gamma^+ a_\delta$ . Therefore

$$A_{\alpha\beta}(\omega) = \frac{\langle a_\gamma^+ a_\delta \rangle}{\hbar\bar{\omega} + (E_\gamma - E_\delta) + \Omega_{\alpha\beta} - \Gamma_{\alpha\beta}(\omega)} \quad (3.8)$$

where the lineshape functions  $\Omega$  and  $\Gamma(\omega)$  are defined as

$$\Omega_{\alpha\beta} \equiv \langle L_v a_\gamma^+ a_\delta \rangle / \langle a_\gamma^+ a_\delta \rangle \quad (3.9)$$

$$\Gamma_{\alpha\beta}(\omega) \equiv \langle L_{eq} Q (\hbar\bar{\omega} + L_{eq} Q)^{-1} L_v a_\gamma^+ a_\delta \rangle / \langle a_\gamma^+ a_\delta \rangle. \quad (3.10)$$

In the next section, we will use equation (3.10) in order to derive the absorption coefficient in a system of electrons interacting with longitudinal optical (LO) phonons.

#### 4. Lineshape function for an electron–phonon system in a quantum well

We consider a system of electrons interacting with phonons confined in the quantum well. The equilibrium Hamiltonian  $H_{eq} = H_d + V$  is given by

$$H_d = H_e + H_p = \sum_{\alpha} \langle \alpha | h_0 | \alpha \rangle a_\alpha^+ a_\alpha + \sum_q \hbar\omega_q b_q^+ b_q \quad (4.1)$$

$$V = \sum_q \sum_{\alpha,\mu} C_{\alpha,\mu}(q) a_\alpha^+ a_\mu (b_q + b_{-q}^+) \quad (4.2)$$

where

$$C_{\alpha,\mu}(q) = V_q \langle \alpha | \exp(i\mathbf{q} \cdot \mathbf{r}) | \mu \rangle. \quad (4.3)$$

Here,  $b_q^+$  ( $b_q$ ) denotes the creation (annihilation) operators for a phonon in the state  $|q\rangle \equiv |q, s\rangle$ , where  $q$  and  $s$ , respectively, are the phonon wavevector and polarization index;  $\hbar\omega_q$  is the phonon energy,  $C_{\alpha,\mu}(q)$  is the electron–phonon interaction matrix element,  $V_q$  is the coupling factor which depends on the mode of the phonons, and  $\mathbf{r}$  is the electron position vector. The distance between the barriers, assumed infinitely high, is  $L_z$ . Assuming that the wavefunction vanishes at  $z = 0$  and  $L_z$ , the eigenfunction  $\Psi_\alpha(\mathbf{r})$  and the corresponding eigenvalue  $E_\alpha$  of the single-electron Hamiltonian  $h_0$  are given by

$$\Psi_\alpha(\mathbf{r}) \equiv \Psi_{njk_\parallel}(\mathbf{r}) = u_{nk}(\mathbf{r}) \exp(i\mathbf{k}_\parallel \cdot \mathbf{r}_\parallel) \Phi_{nj}(z) / \sqrt{A} \quad (4.4)$$

$$\Phi_{nj}(z) = \sqrt{2/L_z} \sin(j\pi z/L_z), \quad j = 1, 2, 3, \dots \quad (4.5)$$

$$E_\alpha \equiv E_{njk_\parallel} = E_0 j^2 + k_\parallel^2 \hbar^2 / 2m_n \quad (4.6)$$

where  $E_0 \equiv \hbar^2 \pi^2 / 2m_n L_z^2$ ,  $A$  is the interface area of the sample,  $u_{nk}(\mathbf{r})$  is the periodic part of the bulk Bloch function,  $n$  is the band index,  $\mathbf{k}_\parallel$  and  $\mathbf{r}_\parallel$  are the wavevector and position vector parallel to the well interface, respectively.  $m_n$  is the effective mass in the band  $n$ .

In order to calculate equations (3.9) and (3.10) further, we make use of the following relation:

$$T_R\{\rho_{eq}(H)[L_Q X, a_\alpha^+ a_\beta]\} = T_R\{\rho_{eq}(H)[L_v a_\alpha^+ a_\beta, X]\} - T_R\{\rho_{eq}(H)[L_v P X, a_\alpha^+ a_\beta]\} \quad (4.7)$$

where we have taken into account of the fact that  $\rho_{eq}(H)$  commutes with  $H$  and  $T_R\{ABC\} = T_R\{BCA\}$ . Thus equation (3.10) becomes

$$\Gamma_{\alpha\beta}(\omega) = T_R\{\rho_{eq}(H)[L_v a_\alpha^+ a_\beta, (\hbar\bar{\omega} + L_{eq} Q)^{-1} L_v a_\gamma^+ a_\delta]\} / \langle a_\gamma^+ a_\delta \rangle - \langle L_v a_\gamma^+ a_\delta \rangle T_R\{\rho_{eq}(H)[(\hbar\bar{\omega} + L_{eq} Q)^{-1} L_v a_\gamma^+ a_\delta, a_\alpha^+ a_\beta]\} / \langle a_\gamma^+ a_\delta \rangle^2. \quad (4.8)$$

Now we will make the approximation that  $\rho_{eq}(H) \approx \rho_{eq}(H_d)$ , assuming that the interaction is quite weak. Then

$$\Omega_{\alpha\beta} = 0 \quad (4.9)$$

because the ensemble averages of  $b_q$  and  $b_q^+$  are zero.

$$\Gamma_{\alpha\beta}(\omega) \approx T_R\{\rho(H_d)[L_v a_\alpha^+ a_\beta, (\hbar\bar{\omega} + L_d)^{-1} L_v a_\gamma^+ a_\delta]\} / \langle a_\gamma^+ a_\delta \rangle \quad (4.10)$$

in the approximation to second order of the scattering potential.

In equations (3.8) and (3.10), the quantity  $\langle a_\gamma^+ a_\delta \rangle$  is given by

$$\langle a_\gamma^+ a_\delta \rangle \equiv T_R\{\rho_{eq}[a_\gamma^+ a_\delta, a_\alpha^+ a_\beta]\} = (f_\beta - f_\alpha) \delta_{\gamma\beta} \delta_{\delta\alpha}. \quad (4.11)$$

Here  $f_\alpha = 1/[1 + \exp\{(E_\alpha - E_F)/k_B T\}]$  is the Fermi–Dirac distribution function for electrons, where  $E_F$  is the quasi-Fermi level of conduction or the valence band.

Therefore, using equations (3.2), (3.8) and (4.9)–(4.11), we obtain

$$G = -\frac{\omega}{n_r} \sqrt{\frac{\mu_0}{\epsilon_0}} \text{Im} \sum_{\alpha,\beta} |(R)_{\alpha\beta}|^2 \frac{f_\beta - f_\alpha}{\hbar\bar{\omega} + (E_\beta - E_\alpha) - \Gamma_{\alpha\beta}(\omega)} \quad (4.12)$$

where

$$\Gamma_{\alpha\beta}(\omega) \approx T_R\{\rho_{eq}[L_v a_\alpha^+ a_\beta, (\hbar\bar{\omega} + L_d)^{-1} L_v a_\beta^+ a_\alpha]\} / (f_\beta - f_\alpha). \quad (4.13)$$

After systematic calculation using equations (4.1) and (4.2), we obtain

$$L_v a_\alpha^+ a_\beta = \sum_q \sum_\gamma (b_q + b_{-q}^+) [C_{\gamma,\alpha}(q) a_\gamma^+ a_\beta - C_{\beta,\gamma}(q) a_\alpha^+ a_\gamma] \quad (4.14)$$

$$L_d b_q a_\alpha^+ a_\beta = (E_\alpha - E_\beta - \hbar\omega_q) b_q a_\alpha^+ a_\beta \quad (4.15)$$

$$L_d b_{-q}^+ a_\alpha^+ a_\beta = (E_\alpha - E_\beta + \hbar\omega_{-q}) b_{-q}^+ a_\alpha^+ a_\beta. \quad (4.16)$$

Inserting these equations into equation (4.13), we obtain

$$\begin{aligned}
 &\Gamma_{\alpha\beta}(\omega)(f_\alpha - f_\beta) \\
 &= \sum_q \sum_\gamma |C_{\beta,\gamma}(q)|^2 \left[ \frac{(1 + N_q)f_\alpha(1 - f_\gamma)}{\hbar\bar{\omega} + (E_\gamma - E_\alpha) + \hbar\omega_q} - \frac{N_q f_\gamma(1 - f_\alpha)}{\hbar\bar{\omega} + (E_\gamma - E_\alpha) + \hbar\omega_q} \right. \\
 &\quad \left. + \frac{N_q f_\alpha(1 - f_\gamma)}{\hbar\bar{\omega} + (E_\gamma - E_\alpha) - \hbar\omega_q} - \frac{(1 + N_q)f_\gamma(1 - f_\alpha)}{\hbar\bar{\omega} + (E_\gamma - E_\alpha) - \hbar\omega_q} \right] \\
 &\quad + \sum_q \sum_\gamma |C_{\alpha,\gamma}(q)|^2 \left[ \frac{(1 + N_q)f_\gamma(1 - f_\beta)}{\hbar\bar{\omega} + (E_\beta - E_\gamma) + \hbar\omega_q} - \frac{N_q f_\beta(1 - f_\gamma)}{\hbar\bar{\omega} + (E_\beta - E_\gamma) + \hbar\omega_q} \right. \\
 &\quad \left. + \frac{N_q f_\gamma(1 - f_\beta)}{\hbar\bar{\omega} + (E_\beta - E_\gamma) - \hbar\omega_q} - \frac{(1 + N_q)f_\beta(1 - f_\gamma)}{\hbar\bar{\omega} + (E_\beta - E_\gamma) - \hbar\omega_q} \right] \tag{4.17}
 \end{aligned}$$

where  $N_q = 1/[\exp(\hbar\omega_q/k_B T) - 1]$  is the Planck distribution function for a phonon with energy  $\hbar\omega_q$ . Equation (4.17) is our new formula, which is similar to those for some other theories [3, 4], but our result can be interpreted in a more organized way since we have derived the result using the quantum statistical many-body projection technique. The physical interpretation is as follows: the first term represents the transition of the electron from the state  $\alpha$  to the intermediate state  $\gamma$  with phonon emission. Here,  $1 + N_q$  appears as the condition for phonon emission and  $f_\alpha(1 - f_\gamma)$  as that for the transition  $\alpha \rightarrow \gamma$ .  $|C_{\beta\gamma}(q)|^2$  means that the intermediate state  $\gamma$  is disturbed by the interaction with phonons. The energy term  $\hbar\bar{\omega} + E_\gamma - E_\alpha + \hbar\omega_q$  enforces energy conservation, i.e.,  $E_\alpha = \hbar\bar{\omega} + E_\gamma + \hbar\omega_q$ . All seven other terms in equation (4.17) can be explained in a similar manner.

Therefore, inserting equation (4.17) into (4.12), the absorption coefficient can be obtained as

$$G = -\frac{\omega}{n_r} \sqrt{\frac{\mu_0}{\epsilon_0}} \sum_{\alpha,\beta} |(R)_{\alpha\beta}|^2 \frac{(f_\beta - f_\alpha)\gamma_{\beta\alpha}(\omega)}{[\hbar\omega + (E_\beta - E_\alpha) - \Delta_{\beta\alpha}(\omega)]^2 + [\gamma_{\beta\alpha}(\omega)]^2}. \tag{4.18}$$

Here the lineshift function  $\Delta_{\beta\alpha}(\omega)$  and the linewidth function  $\gamma_{\beta\alpha}(\omega)$  are defined as

$$\begin{aligned}
 \Delta_{\beta\alpha}(\omega)(f_\alpha - f_\beta) &\equiv \text{Re}\{\Gamma_{\alpha\beta}(\bar{\omega})(f_\alpha - f_\beta)\} = \sum_q \sum_\gamma |C_{\alpha\gamma}(q)|^2 \left\{ [(1 + N_q)f_\gamma(1 - f_\beta) \right. \\
 &\quad \left. - N_q f_\beta(1 - f_\gamma)] \text{P}\left(\frac{1}{\hbar\omega + (E_\beta - E_\gamma) + \hbar\omega_q}\right) \right. \\
 &\quad \left. + [N_q f_\gamma(1 - f_\beta) - (1 + N_q)f_\beta(1 - f_\gamma)] \text{P}\left(\frac{1}{\hbar\omega + (E_\beta - E_\gamma) - \hbar\omega_q}\right) \right\} \\
 &\quad + \sum_q \sum_\gamma |C_{\beta\gamma}(q)|^2 \left\{ [(1 + N_q)f_\alpha(1 - f_\gamma) \right. \\
 &\quad \left. - N_q f_\gamma(1 - f_\alpha)] \text{P}\left(\frac{1}{\hbar\omega + (E_\gamma - E_\alpha) + \hbar\omega_q}\right) \right. \\
 &\quad \left. + [N_q f_\alpha(1 - f_\gamma) - (1 + N_q)f_\gamma(1 - f_\alpha)] \text{P}\left(\frac{1}{\hbar\omega + (E_\gamma - E_\alpha) - \hbar\omega_q}\right) \right\} \tag{4.19}
 \end{aligned}$$

and

$$\begin{aligned}
 \gamma_{\beta\alpha}(\omega)(f_\alpha - f_\beta) &\equiv \text{Re}\{\Gamma_{\alpha\beta}(\bar{\omega})(f_\alpha - f_\beta)\} = \pi \sum_q \sum_\gamma |C_{\alpha\gamma}(q)|^2 \{ [(1 + N_q)f_\gamma(1 - f_\beta) \\
 &\quad - N_q f_\beta(1 - f_\gamma)] \delta(\hbar\omega + (E_\beta - E_\gamma) + \hbar\omega_q) \\
 &\quad + [N_q f_\gamma(1 - f_\beta) - (1 + N_q)f_\beta(1 - f_\gamma)] \delta(\hbar\omega + (E_\beta - E_\gamma) - \hbar\omega_q) \}
 \end{aligned}$$

$$\begin{aligned}
& + \pi \sum_q \sum_\gamma |C_{\beta\gamma}(q)|^2 \{[(1 + N_q)f_\alpha(1 - f_\gamma) \\
& - N_q f_\gamma(1 - f_\alpha)]\delta(\hbar\omega + (E_\gamma - E_\alpha) + \hbar\omega_q) \\
& + [N_q f_\alpha(1 - f_\gamma) - (1 + N_q)f_\gamma(1 - f_\alpha)]\delta(\hbar\omega + (E_\gamma - E_\alpha) - \hbar\omega_q)\}. \quad (4.20)
\end{aligned}$$

Here we have used the Dirac identity  $\lim_{s \rightarrow 0^+} (x + is)^{-1} = P(1/x) - i\pi\delta(x)$ , where ‘P’ means Cauchy’s principal value.

## 5. Lorentzian approximation and relaxation rate

In this section, we assume that  $\Delta_{\beta\alpha}(\omega)$  can be neglected in comparison with  $E_\alpha - E_\beta$  because we consider a very weak scattering effect and we adopt the approximation that  $\gamma_{\beta\alpha}(\omega) \approx \gamma_{\beta\alpha}(E_\alpha - E_\beta) = \gamma_{\beta\alpha}(E_{\alpha\beta})$ , i.e., we assume that it is a very slowly varying function of  $\omega$  near the resonance point ( $\omega = E_\alpha - E_\beta$ ). With these approximations, from equation (4.18) the absorption coefficient, equation (4.18), is given by

$$G = \frac{\omega}{n_\gamma} \sqrt{\frac{\mu_0}{\epsilon_0}} \sum_{\alpha,\beta} |(R)_{\alpha\beta}|^2 \frac{(f_\alpha - f_\beta)\gamma_{\beta\alpha}(E_{\alpha\beta})}{(\hbar\omega - E_{\alpha\beta})^2 + [\gamma_{\beta\alpha}(E_{\alpha\beta})]^2}. \quad (5.1)$$

Here, from equation (4.20) we have the halfwidth  $\gamma_{\beta\alpha}(E_{\alpha\beta})$ :

$$\begin{aligned}
\gamma_{\beta\alpha}(E_{\alpha\beta})(f_\alpha - f_\beta) & = \pi \sum_q \sum_\gamma |C_{\alpha\gamma}(q)|^2 \{[(1 + N_q)f_\gamma(1 - f_\beta) \\
& - N_q f_\beta(1 - f_\gamma)]\delta(E_\alpha - E_\gamma + \hbar\omega_q) \\
& + [N_q f_\gamma(1 - f_\beta) - (1 + N_q)f_\beta(1 - f_\gamma)]\delta(E_\alpha - E_\gamma - \hbar\omega_q)\} \\
& + \pi \sum_q \sum_\gamma |C_{\beta\gamma}(q)|^2 \{[(1 + N_q)f_\alpha(1 - f_\gamma) \\
& - N_q f_\gamma(1 - f_\alpha)]\delta(E_\gamma - E_\beta + \hbar\omega_q) \\
& + [N_q f_\alpha(1 - f_\gamma) - (1 + N_q)f_\gamma(1 - f_\alpha)]\delta(E_\gamma - E_\beta - \hbar\omega_q)\}. \quad (5.2)
\end{aligned}$$

In order to derive the interband relaxation rate, first the subscripts  $\alpha$  and  $\beta$  are rewritten as  $cjk_\parallel$  and  $vj'k'_\parallel$ , where  $c$  and  $v$  refer to conduction and valence bands, respectively.  $j$  and  $j'$  are the subband numbers of a quantum well structure, and  $k_\parallel$  and  $k'_\parallel$  are the wavevectors parallel to the well interface. In the matrix elements  $(R)_{\alpha\beta}$ , we assume that the selection rules  $j = j'$  and  $k_\parallel = k'_\parallel$  hold for the transition between the levels  $\alpha$  and  $\beta$  [18]. Then we have

$$\begin{aligned}
\gamma_{cv}(E_{cv}) & = \pi \sum_q \sum_{nj_n k_\parallel} |C_{cn}(q)|^2 [f(E_{vj_k_\parallel}) - f(E_{cj_k_\parallel})] \\
& \times \{[(1 + N_q)f(E_{nj_n k_\parallel^n})(1 - f(E_{vj_k_\parallel})) \\
& - N_q f(E_{vj_k_\parallel})(1 - f(E_{nj_n k_\parallel^n}))]\delta(E_{cj_k_\parallel} - E_{nj_n k_\parallel^n} + \hbar\omega_q) \\
& + [N_q f(E_{nj_n k_\parallel^n})(1 - f(E_{vj_k_\parallel})) - (1 - N_q)f(E_{vj_k_\parallel})(1 - f(E_{nj_n k_\parallel^n}))] \\
& \times \delta(E_{cj_k_\parallel} - E_{nj_n k_\parallel^n} - \hbar\omega_q)\} \\
& + \pi \sum_q \sum_{nj_n k_\parallel} |C_{vn}(q)|^2 [f(E_{vj_k_\parallel}) - f(E_{cj_k_\parallel})] \\
& \times \{[(1 + N_q)f(E_{cj_k_\parallel})(1 - f(E_{nj_n k_\parallel^n})) - N_q f(E_{nj_n k_\parallel^n})(1 - f(E_{cj_k_\parallel}))] \\
& \times \delta(E_{nj_n k_\parallel^n} - E_{vj_k_\parallel} + \hbar\omega_q) \\
& + [N_q f(E_{cj_k_\parallel})(1 - f(E_{nj_n k_\parallel^n})) - (1 - N_q)f(E_{nj_n k_\parallel^n})(1 - f(E_{cj_k_\parallel}))] \\
& \times \delta(E_{nj_n k_\parallel^n} - E_{cj_k_\parallel} - \hbar\omega_q)\}. \quad (5.3)
\end{aligned}$$



Here  $n = c$  or  $v$  and

$$|C_{\alpha n}(q)|^2 = |V_q \langle \alpha | \exp(i\mathbf{q} \cdot \mathbf{r}) | n \rangle|^2 \\ = V_q^2 \delta(\mathbf{k}_{\parallel}, \mathbf{k}_{\parallel}^n + \mathbf{q}_{\parallel}^n) \delta_{\alpha n} \left| \int \Phi_{nj}(z) \exp(iq_z z) \Phi_{nj_n}(z) dz \right|^2. \quad (5.4)$$

For the interaction matrix element  $C_{\beta\alpha}(q)$ , we take into account the screening effect as follows:

$$V_q^2 = \frac{e^2 \hbar \omega_l}{2\epsilon_0 \Omega} \left( \frac{1}{\epsilon(\infty)} - \frac{1}{\epsilon(0)} \right) \frac{q^2}{(q^2 + q_d^2)^2} \quad (5.5)$$

where  $\Omega$  is the volume of the system,  $\epsilon(\infty)$  and  $\epsilon(0)$ , respectively, are the optical and static dielectric constants,  $\hbar\omega_l$  the energy of the LO phonon,  $q_d$  the reciprocal of the Debye screening length.

## 6. Numerical result

The linewidths for the interband transition in the GaAs/Ga<sub>1-x</sub>As<sub>x</sub>Al quantum well will be calculated numerically for the one-subband case ( $j = 1$  and  $j_n = 1$ ) at the subband edge ( $k_{\parallel} = 0$ ). For that purpose, considering the delta functions in equations (5.3) and (5.4), we obtain

$$\gamma_{cv}(E_{cv}) = \frac{1}{4\pi\hbar^2} \sqrt{\frac{\hbar m_c}{2\omega_l}} \int dq_z |V_c(q)|^2 q_{\parallel}^c / [f(E_{v10}) - f(E_{c10})] \\ \times \{[(1 + N_q) f(E_{c1q_{\parallel}^c}) (1 - f(E_{v10})) - N_q f(E_{v10}) (1 - f(E_{c1q_{\parallel}^c}))]\} \\ + \frac{1}{4\pi\hbar^2} \sqrt{\frac{\hbar m_h}{2\omega_l}} \int dq_z |V_v(q)|^2 q_{\parallel}^v / [f(E_{v10}) - f(E_{c10})] \\ \times \{[(1 + N_q) f(E_{c10}) (1 - f(E_{v1q_{\parallel}^v})) - N_q f(E_{v1q_{\parallel}^v}) (1 - f(E_{c10}))]\}. \quad (6.1)$$

Here

$$f(E_{nj k_{\parallel}}) = \frac{1}{1 + \exp[(E_{nj k_{\parallel}} - E_{fn}) / k_B T]} \quad (6.2)$$

where

$$E_{n1q_{\parallel}^n} = \frac{\hbar^2 \pi^2}{2m_n L_z^2} + \frac{(\hbar q_{\parallel}^n)^2}{2m_n} \quad (6.3)$$

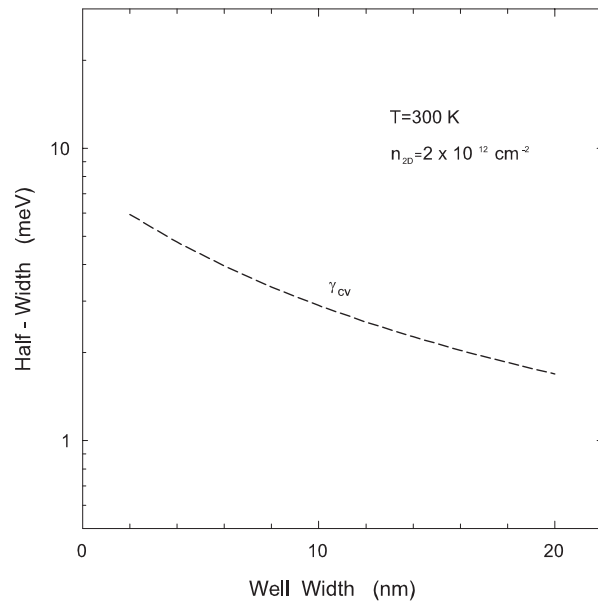
$$q_{\parallel}^n = (1/\hbar) \sqrt{2m_n \hbar \omega_l}. \quad (6.4)$$

The parameters are as follows [19, 20]:

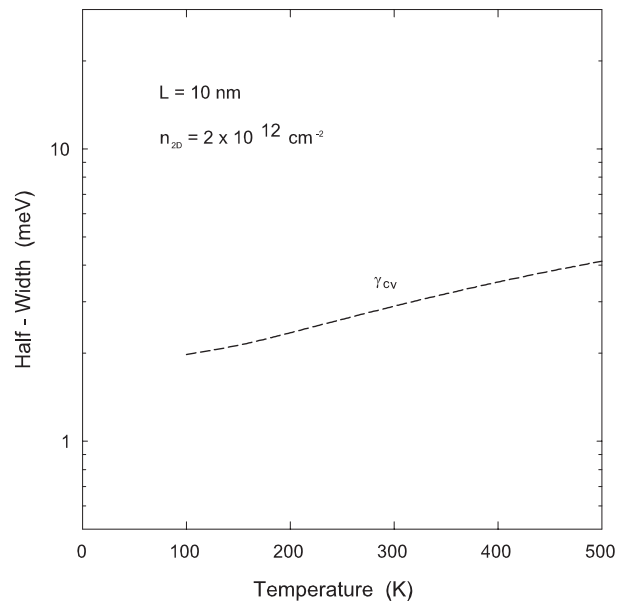
$$m_c = 0.067 m_0, \quad m_{lh} = 0.087 m_0, \quad m_{hh} = 0.5 m_0, \quad \epsilon(0) = 12.53, \\ \epsilon(\infty) = 10.90, \quad E_g = 1.424 \text{ eV}, \quad \omega_l = 2\pi \times 8.76 \times 10^{12} \text{ Hz}$$

where  $m_0$  is the free electron mass.

Figure 1 shows the well width dependence of the halfwidths for the one-subband case ( $j = j_n = 1$ ) at the subband edges ( $k_{\parallel} = 0$ ) for GaAs/Ga<sub>1-x</sub>As<sub>x</sub>Al quantum well structures. As we can see, the width decreases with the well width, which is physical. Note that the scattering is reduced as the dimension is increased in general, as long as no other potential is involved. Figure 2 shows the temperature dependence of the halfwidth, which is also physical. Note that the same tendency appears in cyclotron transitions and interband magneto-optical transitions [14]. We see that the widths increase with the temperature. We can interpret this increase as being caused by the phonon distribution.



**Figure 1.** The well width dependence of the halfwidth at the subband edges for GaAs/Ga<sub>1-x</sub>As<sub>x</sub> Al.



**Figure 2.** The temperature dependence of the halfwidth at the subband edges for GaAs/Ga<sub>1-x</sub>As<sub>x</sub> Al.

## 7. Concluding remarks

We have derived a new absorption coefficient formula for the optical transition in a system of electrons interacting with phonons by using the operator algebra technique. The lineshape

function appearing in the absorption coefficient is similar to those in some other theories [3, 5], but our result contains the electron and phonon distribution more phenomenologically. Therefore, all the possible processes of electron transitions along with phonon absorptions and emissions could be explained properly in compliance with the energy conservation law and the conditions for optical transitions in solids. The numerical results show that the width decreases with the well width and increases with the temperature, as we could easily guess. It is to be regretted that the result cannot be checked in a satisfactory way, since no experimental data are available for this problem. Studies on electron–electron interaction and the second-order term in equation (2.12), which gives us the Raman interaction [21], will be performed in the future.

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